Market Making in NFTs

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Abstract

This study discusses a framework for market making in Non-Fungible Tokens (NFTs) which represent unique digital assets in a low liquidity and high price volatility environment. The market making problem differs from fungible cryptocurrencies as there is no layered orderbook structure of market depth and market makers would need to hold inventory on the other side of the trades for a non-negligible horizon. We have developed a model for prices in market making context in terms of embedded optionality of trading an NFT at either the floor or a price that is closer to the appraisal value. In order to develop the model for optionality of trading an NFT, we have defined a model on the joint price dynamics of the NFT and its corresponding floor. The price dynamics were decomposed into an intrinsic price diffusion process and jump components relating to liquidity events. Besides NFTs, the approach presented in this paper could provide insights for market making in other "non-fungible" asset classes such as art or housing.

Keywords: NFTs, Digital Assets, Market Making, Market Microstructure

1 Introduction

The Non-Fungible Token (NFTs) market has been one of the most rapidly growing segments within the digital asset industry. In 2021, NFT sales volume exceeded 25 billion USD with weekly trading volume reaching up to a billion dollar [5]. Despite the large growth and volume, there has been limited research on market making in NFTs and most crypto market makers have not yet ventured into market making of the non-fungible counterparts. The professional trading ecosystem is yet to be properly researched and developed.

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One could view market making as a series of actions where an equilibrium is found that balances supply and demand. Traditionally market makers provide inventory and search cost reduction services. Market making can thus be difficult for low liquidity assets as traders face larger inventory risk and higher search costs. The NFT market presents a low liquidity and high price volatility environment with long potential holding times to sell near fair appraisal prices. NFT market makers would thus need to hold a volatile asset in their inventory with the risk of significant downward price movements while trying to find an appropriate buyer. This uncertainty can cause prices which market makers are willing to offer to deviate significantly from appraisal values as steep discounts are charged to cover the risks. The market making problem differs from fungible tokens in two aspects. First, each token is considered to be unique albeit driven by certain lower-dimensional systematic factors. Hence, it is difficult to create a layered orderbook structure of market depth. Secondly, given the illiquidity there is a relatively long expected duration between transactions. In this case, liquidity providers would need to hold inventory on the other side of trades for a non-negligible horizon with the objective of longer-duration alpha creation on the inventory portfolio rather than market-neutral spreads.

We have developed a model for prices in market making context in terms of embedded optionality of trading an NFT. In this regard we have considered the value and time preference of executing a trade as a function of price volatility, correlation, and liquidity. In order to develop the model for optionality of trading an NFT, we have defined a model on the joint price dynamics of the NFT and its corresponding floor. The price dynamics were decomposed into an intrinsic price diffusion process and jump components relating to liquidity events. Liquidity episodes tend to exhibit clustering due to contagion in both market fundamentals and sentiment where we have modeled the sentiment dynamics as a Hawkes process. This enables us to produce jump clustering behavior that is appropriate for a complementary liquidity process. Using this price process, we have employed an option pricing methodology to value the optionality for the market maker as a trade-off between accepting the immediate floor or taking a chance for a higher price.

Our design contributes to the market microstructure literature in two ways. Firstly, we provide a framework for market making in NFTs. Research within digital asset trading is still in an early stage, and to the best of our knowledge, this is the first paper on market making in NFTs. Secondly, we provide a framework for market making in illiquid and unique assets. There is limited research on the topic and these insights could be used for trading in assets such as art, housing and luxury items.

2 Background

2.1 NFT Market

An NFT is a non-interchangeable unit of data stored on the blockchain. The main difference relative to fungible crypto tokens is that NFTs represent a single, unique, and indivisible crypto asset. The largest portion of NFT trading volume is concentrated in collections where numerous NFTs with heterogeneous characteristics are released as part of an encompassing collection [19]. For example, popular collections include Crypto Punks and Azuki. The number of NFTs per collection varies, for instance, the above-mentioned collections contain 10.000 NFTs. In collection-based NFTs, holders receive a base utility from exposure to the collection and idiosyncratic utility captured by the specific features.

The NFT market has reached almost 25 billion USD in sales in 2021 from less than 1 billion USD one year earlier. NFT trading has exponentially grown in the past year, and further growth could be expected as NFTs have been rapidly gaining adoption in numerous financial and non-financial use cases. The primary sale of NFTs can happen through a direct sale of pre-minted NFTs from the creator or in so-called mints where people mint tokens via a smart contract. Secondary trading has been mainly performed on peer-to-peer exchanges. These exchanges act like marketplaces where people can list and bid on each other's NFTs. NFTs have been mainly traded on centralized exchanges while decentralized exchanges — which have been popular trading venues for fungible cryptocurrencies — have just started making their way into the ecosystem.

Despite the growth in trading volume, NFTs remain considerably illiquid. For example, analyzing NFT collections on OpenSea shows that 39 % of the NFTs have never been sold after the mint and 92 % have only been sold three times or less. The NFT market presents a low liquidity and high price volatility environment. This illiquid and volatile nature of NFTs can present a challenge for market makers as they face large inventory risk and high potential search costs.

2.2 Market Making in Illiquid Assets

One could view market making as a series of actions where an equilibrium is found that balances supply and demand [13]. As argued by Dolgopolov [7], the crux of market making entails the provision of liquidity in assets by committing own capital. Traditionally, market makers provide inventory and help determine prices by matching demand and supply in an auction-like game. Market makers essentially provide inventory and search cost reduction services for which a (risk-adjusted) fee is charged in terms of bid-ask spreads.

Conventional market microstructure theory predicts that market makers require substantially lower prices when there are fewer buyers, longer holding times and higher asset price volatility [12]. Market making can be relatively difficult in less liquid markets as traders have an incentive not to post orders as it would reveal information with little chance of any benefit relating to the no-trade theorem of Milgram and Stokey [4]. In the case of illiquid assets market makers tend to prefer to rather act like intermediaries who match demand and supply [7].

D'Arcy (2005) [1] argues that real estate service providers act as "market makers" in the housing market as they facilitate information sharing and reduce information costs. In this context one could view aggregators as "market makers" in the NFT market as they facilitate information across market places to help link buyers and sellers. However, we would like to argue that market makers not only facilitate information but directly facilitate liquidity. There have been numerous researchers (e.g., Bayer et al. [3]) who empirically investigated real estate flipping which could be considered as a form of market making. For example, Agarwal et al. [2] show that real estate agents use their informational advantage and bargaining power in order to buy at discounted prices. However, real estate agents would rather cherry pick and use bargaining power over weak sellers which could hardly be argued as systematic market making activity. Bayer et al. [3] argue the existence of market making middlemen who purchase housing below market value with the goal of reselling quickly. These types would possess strong "deal picking" skills and would buy from "motivated" sellers in need of liquidity. Zillow — a tech company that created an online market place for real estate in the US — attempted to introduce some form of market making within the real estate using pricing algorithms. However, their pricing algorithms failed and Zillow had to shut down its house flipping operations after steep losses. This is related to the aforementioned argument that market makers would need to hold inventory for a non-negligible horizon with an objective of longer-duration alpha creation on the inventory which can be risky.

Lovo and Spaenjers [18] develop a model for trading in art auction markets where they argue that traders take into account the expected resale value and the value of the "emotional dividend" while holding the art piece. Within the art market, one could argue that the large network of individual dealer facilitates the role of market making as art is bought and sold in the hope to generate profits. The lack of research on market making models in art could be explained by the lack of data availability, the relatively obscure nature of the market, and the difficulties in pricing unique art pieces. Furthermore, there are various market imperfections such as authentication of art works and proof of ownership which makes market making difficult. Art dealers tend to provide the function of experts who authenticate the art works which can be a time-intensive process.

3 NFT Market Making

The illiquid and volatile nature is not necessarily the biggest issue but the fact that NFTs represent unique assets. As a result, it is difficult to create a layered order book structure as in other fungible assets. As previously discussed, an NFT will be valued differently by various potential buyers depending on their private utility function. The general value of the collection is roughly shared among participants while the idiosyncratic value of the NFT could be valued differently. Market makers would be required to hold a volatile asset in their inventory with the risk of significant downward price movements while trying to find an appropriate buyer. Given the market making risks, liquidity providers would offer prices closer to the floor than the appraisal price unless they expect the NFT to be desirable with low potential search costs. This could help explain the observation by [15] that a relatively large fraction of NFTs within a collection trade near the floor. There is a limited literature on market making in unique assets and one could argue similar issues in the art and real estate market due to the heterogeneous nature of these assets.

The market making problem in NFTs differs from ”fungible” assets in two aspects. Firstly, each asset is considered to be unique albeit driven by certain lower-dimensional features. Hence, there is no layered orderbook structure of market depth. Secondly, because of the illiquidity there is a long expected duration between transactions. Liquidity providers would need to hold inventory on the other side of the trades for a non-negligible and rolling horizon with the objective of longer-duration alpha creation on the inventory portfolio. This differs from conventional market making models where market neutral positions tend to be the norm. As a result, the market making framework would need to reflect these realities.

Due to the uniqueness of NFTs there are high potential search costs to find an appropriate buyer who is willing to pay a ”fair” price. The floor of a collection represents the minimum price at which NFTs within the collection could be sold while the price component in excess of the floor represents the idiosyncratic utility of the NFT in question. The floor price could be considered as the liquid part where there are few search costs to find willing buyers at this price. On the other hand, the idiosyncratic feature-based pricing element is the more illiquid part with high potential search costs to find appropriate buyers where one would need to potentially accept a (considerable) liquidity discount in order to sell. Taking inspiration from ABBBBBB NFT (2022), one could conceptually view NFT liquidity — in terms of search time and costs — as illustrated in figure 1. This is related to Duffie, Gârleanu, and Pedersen [8] who show that higher search costs cause worse prices in over-the-counter markets. Desirable NFTs — such as ones with popular or rare characteristics — tend to be significantly more liquid than average or lower tier NFTs. Search costs are relatively low with desirable NFTs as it is easier to find appropriate buyers willing to pay the potential price. In the case of undesirable NFTs it is difficult to receive a fair feature-based price and one would need to settle near the floor. It should be emphasized that these curves are a conceptual representation based on discretionary and expert knowledge of the NFT market.

Estimating these curves requires extensive listing and bidding data which is left as an avenue for future research.

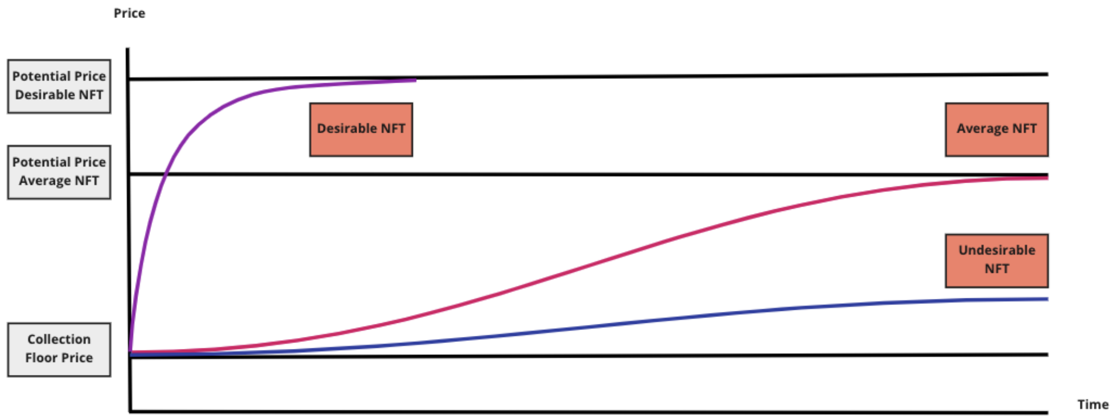

Figure 1: Price and Holding Period

4 Pricing Model for NFT Market Making

We have developed a model for fair prices in market making context where we view the problem in terms of optionality of trading an NFT at either the floor or taking a chance to sell at a higher price that is closer to the fair appraisal price. In order to develop a model for embedded optionality of trading an NFT, we first defined a model on the joint price dynamics of the NFT and its corresponding floor. It should be emphasized that our approach applies to collection-based NFTs where there are a number of heterogeneous NFTs within a collection. As argued by Oh, Rosen and Zhang [19] the largest fraction of NFT trading volume is concentrated in collections so the model would be applicable to the largest part of the NFT market.

4.1 Price Dynamics

Collection-based NFTs have significant exposure to the collection they are part of. The collection could be seen as the "family" an NFT belongs to where there is strong systematic co-movement between different NFTs within that collection. The floor price of a collection represents the lower bound at which NFTs within the collection are traded. It can be argued to represent the market price of immediate liquidity of any NFT within the collection and a measure for the base value of the collection. The price in excess of the floor could be argued to represent the idiosyncratic utility of the NFT. NFT prices can represent other elements outside of the collection floor and the idiosyncratic feature-based component such as systematic factors influencing prices.

The price dynamics can be decomposed into an *intrinsic* price diffusion process and jump components attributed to liquidity events. The intrinsic price process models the continuous diffusion of prices from conventional information dissemination and continuous price discovery in financial markets. This is complemented with a jump process that models large liquidity events. Liquidity events represent the arrival of large exogenous events that cause discontinuous price jumps on thinly traded markets with fragmented liquidity. These may include the arrival of unexpected macro or crypto-market-wide news such as regulatory or monetary policy changes, cascading liquidations of large leveraged wallets, sequence of collateral failures, protocol security breaches, momentum ignitions through "pumps", etc. The magnitude of consequent price movements are amplified under lower liquidity conditions. Since the continuous and discontinuous components differ both in induced price action and causal mechanics, we elect to separate the two, but combine the processes multiplicatively in the end to model the *complete* price process.

Liquidity shocks often exhibit clustering effects which can be caused due to common exposure to observable [6] and unobservable factors [9], as well as direct contagion in market-wide events [10]. Market contagion dynamics can be modeled as Hawkes processes which are self-exciting, by which the arrival of jumps further increases the intensity and thus the probability of future jumps. This enables us to produce feedback behavior that is appropriate for the complementary liquidity process, which would not be possible via jump diffusion models based on, for example, doubly stochastic Poisson processes alone. Self-exciting jump dynamics permit both negative contagion as well as discontinuous positive momentum ignition.

Consider a complete probability space $(\Omega, \mathcal{F}, P)$ with respect to a complete and right-continuous information filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$. As previously mentioned, we decompose the complete price process S(t) into an intrinsic process $\tilde{S}(t)$ and liquidity process L(t) in the form of

$$S(t) = \tilde{S}(t) \cdot L(t) \tag{1}$$

at time t. Let us assume there is a riskless yield r generating numeraire asset (either USD with funding rate or ETH with staking yield depending on the vantage point appropriately assumed for the collective liquidity provider economy). Let the *intrinsic price* process $\tilde{S}_k(t)$ for a given NFT and the collection floor, denoted by subscripts i and f respectively, be given by

$$\tilde{S}_k(t) = S_k(0) \exp\left\{ \left(r - \frac{1}{2}\sigma_k^2 \right) t + \sigma_k W_k(t) \right\} \qquad k = i, f \tag{2}$$

where $sigma_k$ represents the volatility, and $W_i(t)$ and $W_f(t)$ are correlated Wiener processes with a correlation coefficient ρ_{if}. Note that the overall multivariate correlation matrix $[P]_{ij}$ can be governed by a richer factor structure of underlying feature vectors. However, for the pricing of a single NFT in the current exposition, we only require an estimate of the pairwise correlation between the NFT and corresponding collection floor.

Now consider the sequences of stopping times defined on the probability space $(\tau_j)_{j\in\mathbb{N}}$, $(\tau_{i,j})_{j\in\mathbb{N}}$, $(\tau_{f,j})_{j\in\mathbb{N}}$ which generate the following nonexplosive counting processes

$$\begin{aligned} N(t) &= \sum_{j=1}^{\infty} \mathbb{I}_{\tau_j \le t} \\ N_k(t) &= \sum_{j=1}^{\infty} \mathbb{I}_{\tau_{k,j} \le t} \qquad k = i, f \end{aligned} \tag{3}$$

The non-subscripted $N(t)$ counts the collection-wide liquidity event arrivals whereas $N_k(t), k = i, f$ represent idiosyncratic liquidity events. We model the overall liquidity events for $k = i, f$ to arrive either through market liquidity events or idiosyncratic liquidity events as

$$\overline{N}_{k,t} = N_t + N_{k,t} \qquad k = i, f \tag{4}$$

These processes can be characterized directly through their conditional arrival rates or intensities λ. Following the work of Errais, Giesecke and Goldberg [10] we propose dynamics in the form of

$$\lambda(t) = c + e^{-\gamma t}(\lambda(0) - c) + \int_0^t \eta e^{-\gamma s} dN(s). \tag{5}$$

With the reversion level set to the initial value $c = \lambda(0)$ we have

$$\begin{aligned} \lambda(t) &= \lambda(0) + \eta \sum_{\tau_j \le t} e^{-\gamma(t-\tau_j)} \\ \lambda_k(t) &= \lambda_i(0) + \eta_i \sum_{\tau_{k,j} \le t} e^{-\gamma_i(t-\tau_{k,j})} \qquad k = i, f \end{aligned} \tag{6}$$

The counting processes of (3) exhibit self-exciting properties as their intensities increase by $\eta, \eta_k, k = i, f$ increments at each jump arrival. Since the arrival of an event immediately increases the conditional arrival probability of its own process, this creates a self reinforcing feedback loop which represents the empirical phenomenon of (liquidity) event clusters. For instance, large liquidations can trigger further liquidations or collateral chains can trigger cascading credit events. However, with the passage of time the environment eventually normalizes such that the impact should be ephemeral over long enough horizons. It should be noted that in the specification of (6), the impact of previous events decays exponentially. The processes specified in (4), characterized by (6) are both self-exciting and mutually-exciting via shared collection-wide process N_t. This construct permits analytical tractability while allowing liquidity co-jumps across the entire collection. Note that in the special case of $\gamma = 0$ these are birth processes and $\delta = 0$ they are regular Poisson processes.

Finally in order to complete the specification, we must define what happens upon liquidity event arrival — i.e. the distribution of price jumps at each event time of $\overline{N}_{k,t}$. For this, we assume that the price jumps follow a log-normal distribution. More specifically, the amplitude of the price jumps at each arrival time of $\overline{N}_{k,t}$ follow a log-normal distribution of $\log\left(1 + Z_{k,j}\right) \sim N(m_k, v_k)$ where we further denote $\mu_k = \exp\left(m_k + \frac{1}{2}v_k^2\right) - 1$. The intrinsic prices experience multiplicative jumps of

$$L_k(t) = A_k(t) \prod_{j=1}^{\overline{N}_{k,t}} \left(1 + Z_{k,j}\right) \qquad k = i, f. \tag{7}$$

Here $A_k(t)$ is needed to counteract the ever-increasing tendency of the counting process and can be derived from the compensator of the jump process $\overline{N}_k(t)$ by which the Doob-Meyer decomposition guarantees that the compensated counting process is a local martingale [14]. By Ito's formula for semimartingales we have

$$\begin{aligned} A_k(t) = \exp\Bigg\{ &- \mu_k(\lambda(0) + \lambda_k(0))t + \sum_{j=1}^{N_t} \frac{\mu_k \eta}{\gamma} \left(e^{-\gamma(t-\tau_j)} - 1\right) \\ &+ \sum_{j=1}^{N_{k,t}} \frac{\mu_k \eta_k}{\gamma_k} \left(e^{-\gamma_k(t-\tau_{k,j})} - 1\right) \Bigg\} \qquad k = i, f. \end{aligned} \tag{8}$$

The *complete price* processes are finally expressed as multiplicative products of the intrinsic process $\tilde{S}_k$ of (2) and the liquidity jumps arriving at $S_k(t) = \tilde{S}_k(t) \cdot L_k(t)$ or

$$S_k(t) = \tilde{S}_k(t) A_k(t) \prod_{j=1}^{\overline{N}_{k,t}} \left(1 + Z_{k,j}\right) \qquad k = i, f. \tag{9}$$

4.2 Pricing the Optionality of Exchange

The market maker's decision problem of optimal price determination can be viewed from the perspective of embedded optionality. For instance, from a seller's perspective as shown in figure 1, a transaction can immediately happen at the floor or one could wait for a potential price improvement that reflects the unique desirability of the NFT. However, since both the market and idiosyncratic drivers of prices are stochastic this has a cost which should reflect both duration and volatility.

The value of the optionality for the market maker can be represented as a time and uncertainty dependent trade-off between accepting the immediate floor or taking a chance for a higher price of ϕ where $\phi \geq 1$ at a future date:

$$V^* = e^{-rT}\mathbb{E}\left[\left(S_i(T) - \phi S_f(T)\right)^+\right] \tag{10}$$

The parameter ϕ becomes an investor or market maker preference parameter. For instance, ϕ can be set as the current exponentially moving average of the theoretical price divided by the floor $\phi = \frac{\bar{S}_i}{S_f}$ or the market maker's estimated multiple according to internal forecasting models. Under the dynamics described in equation (9), this value can be derived, point process pathwise-analytically, on the realized paths of $N_T = n, N_{k,T} = n_k, k = i, f$ in Black-Scholes like form.

Let $X(t) = \log\left(S_f(t)\right)$ and $Y(t) = \log\left(S_i(t)\right) - \log\left(S_f(t)\right)$. We have

$$\begin{aligned}
X(t) &= X(0) + l_f T + \sigma_f W_f(T) + \omega_f + \sum_{j=1}^{n+n_f} \log\left(1 + Z_{f,j}\right) \\
Y(t) &= Y(0) + (l_i - l_f)T + \sigma_i W_i(T) - \sigma_f W_f(T) + \omega_i - \omega_f \\
&\quad + \sum_{j=1}^{n+n_f} \log\left(1 + Z_{f,j}\right) - \sum_{j=1}^{n+n_f} \log\left(1 + Z_{f,j}\right)
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
\omega_k &= \sum_{j=1}^{n} \frac{\mu_k \eta}{\gamma}\left(e^{-\gamma(t-\tau_j)} - 1\right) + \sum_{j=1}^{n_k} \frac{\mu_i \eta_k}{\gamma_k}\left(e^{-\gamma_k(t-\tau_{k,j})} - 1\right) \\
l_k &= r - \mu_k(\lambda(0) + \lambda_k(0)) - \frac{1}{2}\sigma_k^2 \qquad\qquad k = i, f
\end{aligned} \tag{12}$$

Since the sums of normal variables are also normal, $X(T) \sim N(\Theta, \Psi)$, $Y(T) \sim N(\Xi, \Pi)$, and the covariance $Cov\left(X(T)Y(T)\right) = \sigma_i\sigma_f\rho_{if}T - \sigma_f^2 T - \sigma_i^2 T$ where

$$\begin{aligned}
\Theta &= \log\left(S_f(0)\right) + l_f T + \omega_f + (n + n_f)a_f \\
\Xi &= \log\left(S_i(0)\right) - \log\left(S_f(0)\right) + (l_i - l_f)T + \omega_i - \omega_f + (n + n_i)m_i - (n + n_f)m_f \\
\Pi &= \left(\sigma_i^2 + \sigma_f^2\right)T + (n + n_i)v_i + (n + n_f)v_f - 2\sigma_i\sigma_f\rho_{if}T \\
\Psi &= \sigma_f^2 T + (n + n_f)v_f
\end{aligned} \tag{13}$$

Now we can normalize the expressions and write these as

$$\begin{aligned}
X(T) &= \Theta + \sqrt{\Psi}\epsilon_f \\
Y(T) &= \Xi + \sqrt{\Pi}\epsilon_{if}
\end{aligned} \tag{14}$$

where the correlation between the standard normals ϵ_f and ϵ_{if} is given by

$$\xi = \frac{\sigma_i\sigma_f\rho_{if}T - \sigma_f^2 T - (n + n_f)b_f}{\sqrt{\Pi \cdot \Psi}} \tag{15}$$

If $S_k(0) > 0, k = i, f$ then $S_k(t), k = i, f$ are almost surely positive processes by (9). Hence,

$$
\begin{aligned}
\mathbb{E}\left[\left(S_i(T) - \phi S_f(T)\right)^+\right] &= \mathbb{E}\left[S_f(T)\left(\frac{S_i(T)}{S_f(T)} - \phi\right)^+\right] \\
&= \mathbb{E}\left[S_f(T)\right]\mathbb{E}\left[\left(\frac{S_f(T)}{\mathbb{E}\left[S_f(T)\right]}\frac{S_i(T)}{S_f(T)} - \phi\right)^+\right]
\end{aligned}
\tag{16}
$$

The second equality follows from the (14). Now we introduce the floor price Q^f measure define by the Radon-Nikodym derivative $\frac{dQ^f}{dQ} = \frac{S_f(t)}{\mathbb{E}[S_f(t)]}$ under which we can express the second expectation as $\mathbb{E}^f\left[\left(\frac{S_i(T)}{S_f(T)} - \phi\right)^+\right]$. By (14) and (15) $\epsilon_{if} \sim N\left(\xi\sqrt{\Psi}, 1\right)$ under the Q^f measure. Let $\epsilon \sim N(0,1)$ be a standard normal variable, $Z = \exp\left(\Xi + \xi\sqrt{\Pi\cdot\Psi} + \sqrt{\Pi}\epsilon\right)$ and further define

$$
\begin{aligned}
d_- &= \frac{\Xi + \xi\sqrt{\Pi\cdot\Psi} - \log(\phi)}{\sqrt{\Pi}} \\
d_+ &= d_- + \sqrt{\Pi}
\end{aligned}
\tag{17}
$$

then

$$
\begin{aligned}
\mathbb{E}^f\left[\left(\frac{S_i(T)}{S_f(T)} - \phi\right)^+\right] &= \mathbb{E}^f\left[(Z - \phi)^+\right] \\
&= \mathbb{E}^f\left[Z \cdot \mathbb{I}_{Z\leq\phi}\right] - \phi\mathbb{E}^f\left[\mathbb{I}_{Z\geq\phi}\right] \\
&= \exp\left(\Xi + \xi\sqrt{\Pi\cdot\Psi} + \frac{1}{2}\Pi\right)N(d_+) - \phi N(d_-).
\end{aligned}
\tag{18}
$$

By (14), we have $\mathbb{E}\left[S_f(T)\right] = \exp\left(\Theta + \frac{1}{2}\Psi\right)$ and by substituting this in place of the first expectation in (16) we arrive at the Black-Scholes style formulation of the NFT item-floor exchange formulation under dynamic liquidity of

$$
V^* = A\left(BN(d_1) - \phi N(d_2)\right) \tag{19}
$$

where

$$
\begin{aligned}
A &= \exp\left\{-rT + \Theta + \frac{1}{2}\Psi\right\} \\
B &= \exp\left\{\Xi + \xi\sqrt{\Pi\cdot\Psi} + \frac{1}{2}\Pi\right\}
\end{aligned}
\tag{20}
$$

Note that the analytical expression is conditional on the paths of the Hawkes processes, hence must be computed as the sample mean across the simulated paths using an Ogata [16, 21] type thinning algorithm.

4.3 Discussion

The underlying NFT price dynamics specified in the previous section combines *intrinsic price* diffusions with the impact of discontinuous liquidity events in a coherent and parsimonious model. Furthermore, the liquidity events are both self and mutually-exciting, replicating the clustering of price shocks across time and within collections that we empirically observe. This allows us to evaluate the market maker's choice realistically through the perspective of a specific NFT's price with respect to the collection floor and the joint dynamics therein. The additive structure of the Hawkes process can be further expanded for multivariate NFT-economy-wide (or "sector"-wide) models in the future. Constructing realistic yet tractable multivariate dynamics would be beneficial for the NFT market-making use case in which market makers will be required to hold large and diverse inventories for relatively long holding periods.

The framework of viewing the market maker's choice through the option specification of (10) elucidates the different trade-offs at play. The option greeks could be intuitively used to examine the effect of changes in some base decision variables such as Delta (change in collection price), Theta (time), Gamma (effect of collection price changes), Vega (price volatility), Rho (base cost of capital in crypto), etc. The exchange ratio parameter ϕ can be seen as a parameter that represents the market maker's confidence in the unique value of the given NFT in comparison to the collection floor. For instance, one market making strategy could be to use the running average of realized transactions within certain cohorts or to have a trait and rarity based inference model to estimate the appropriate near-the-money ratio. This also interplays with time-preference as a shorter investment horizon impacts the probability of a favorable outcome via theta. The pricing formula of (19) also incorporates the elements such as the impact of correlations, idiosyncratic vs. collection-wide liquidity shocks, etc. The marginal impact and intuition of the parameter sensitivities, as well as rigorous and robust estimation strategies will be left for future work.

The market making model could be used for market making in collection-based NFTs. At the moment there is limited market making activity in NFTs and we provide a framework to think about liquidity provision in this segment. Improving market making could improve trading conditions in a market with little price efficiency and a relatively high degree of illiquidity. This could provide trading efficiency gains, allocation improvements, decreased deadweight loss, and allow for more efficient markets to develop within the NFT space. Market making activity is fundamentally part of efficient asset markets and could almost be seen as the provision of a public good [7]. Automated market makers for NFTs (such as Caviar AMM) could also apply the market making discount as derived from our model to compute prices that are fair for liquidity providers to accept.

4.4 Model Extensions

First of all, it should be noted that in this paper we have presented a structured model how rational market makers could set quote prices. One could apply more statistical or machine learning based models where one would try to estimate for how much they expect to sell the NFT, and set a price (adjusted for a risk spread) based on this. The advantage of our model is that it provides a structured framework on how to view market making, and allows to intuitively tune the parameters.

It should be noted that we did not include a risk premium in this exercise. In our model we assumed a price that market makers would be willing to immediately bid in the market based on price expectations. One could assume that market makers would demand a certain risk premium for taking this intertemporal risk on his books. In expectation, over many trades, this risk premium would approximate the profit margin from market making activity. This risk premium would need to be empirically estimated, and could be constructed as a function of numerous variable representing the risk that the market maker is taking such as liquidity, expected holding time, price volatility, etc.

We have assumed that the floor price is the minimum price that one could get in the market. However, NFT market places (such as Blur) have increasingly allowed for bidding on NFTs. In this case one could take the bids of similar NFTs within the same cluster to compute a "bid floor". This bid floor could be assumed to be the minimum that you could get in the market for the NFT in question. For this exercise you would cluster NFTs within a collection together where the NFTs within a similar cluster could be assumed to be quasi fungible. The dynamics of the model would similar except that you would swap the floor price variable with the bid floor price. We conceptionally illustrate this by figure 2 below.

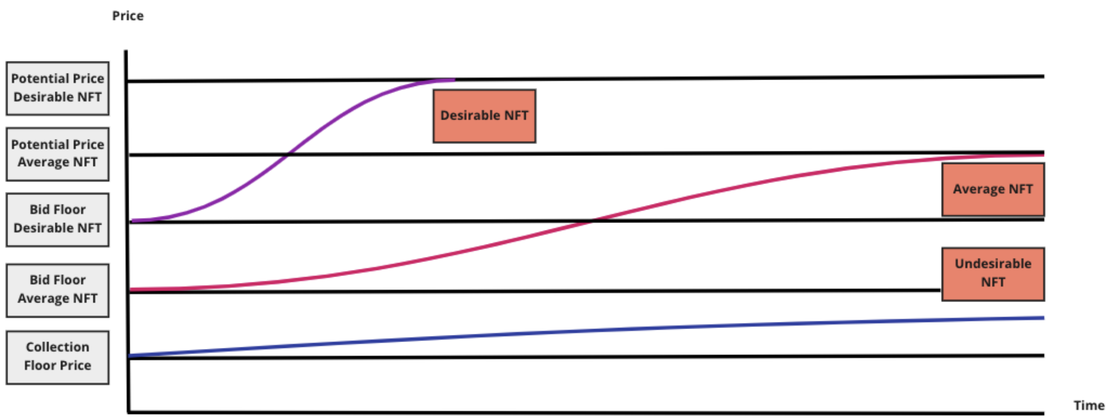

Figure 2: Price and Holding Period

Finally, the framework could be extended to non-collection NFTs in the case that one could generate clusters of NFTs with systematic co-movements. Fundamentally, a collection could be viewed as a cluster of NFTs with strong systematic co-movement to a certain base line (i.e. the collection floor). For example, one could use factor models to structure NFTs in clusters according to a variety of characteristics that would capture systemic co-movements — a cluster of small size NFTs within a certain category, theme and artist network.

5 Conclusion

This study discussed a framework for market making in NFTs. The NFT market represents a low liquidity and high price volatility environment with long potential holding times. This uncertainty about finding appropriate buyers in a relatively short time span can cause transaction prices to deviate significantly from the "fair" appraisal price as discounts are charged to cover market making risks. The market making problem differs from fungible cryptocurrencies as there is no layered orderbook structure of market depth and market makers would need to hold inventory on the other side of the trades for a non-negligible horizon. We have developed a model for market making prices through the lens of optionality of trading an NFT at either the floor or a price closer to the appraisal value. In order to develop the model for embedded optionality of trading an NFT, we first defined a model on the joint price dynamics of the NFT and its corresponding floor. The price dynamics were decomposed into a fundamental price process and jump components relating to liquidity events. The dynamics of the jumps were modeled as Hawkes processes which would enable us to produce jump clustering behavior appropriate for the complementary liquidity process.

Research within digital asset trading is still in an early stage, and we contribute to the literature by providing a framework to think about market making in NFTs. Future research could contribute by further building upon our framework or proposing different conceptual framework to think about the market making problem. Our approach applies to collection-based NFTs where there are a number of heterogeneous NFTs within an overlapping collection. One could potentially expand the market making model for non-collection NFTs in the case that one could generate clusters of NFTs with systematic co-movements. Furthermore, future research could investigate how this conceptual framework or the use of NFTs could be used for art and real estate trading.

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